24th Feb. 2021 | Shift - 1

MATHEMATICS

1. The locus of the mid-point of the line segment joining the focus of the parabola $y^2=4ax$ to a moving point of the parabola, is another parabola whose directrix is:.

(1) $x = a$ (2) $x = 0$ (3) $x = -\frac{a}{2}$ (4) $x = \frac{a}{2}$

Ans. (2)

Sol. $h = \frac{at^2 + a}{2}$, $k = \frac{2at + 0}{2}$

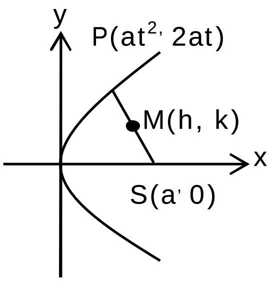
$\Rightarrow t^2 = \frac{2h - a}{a}$ and $t = \frac{k}{a}$

$\Rightarrow \frac{k^2}{a^2} = \frac{2h - a}{a}$

$\Rightarrow$ Locus of (h, k) is $y^2 = a(2x - a)$

$\Rightarrow y^2 = 2a\left(x - \frac{a}{2}\right)$

Its directrix is $x - \frac{a}{2} = -\frac{a}{2} \Rightarrow x = 0$



2. A scientific committee is to formed from 6 Indians and 8 foreigners, which includes at least 2 Indians and double the number of foreigners as Indians. Then the number of ways, the committee can be formed is:

(1) 560 (2) 1050 (3) 1625 (4) 575

Ans. (3)

Sol. (2I, 4F) + (3I, 6F) + (4I, 8F)

$= {}^6C_2\,{}^8C_4 + {}^6C_3\,{}^8C_6 + {}^6C_4\,{}^8C_8$

$= 15 \times 70 + 20 \times 28 + 15 \times 1$

$= 1050 + 560 + 15 = 1625$

3. The equation of the plane passing through the point (1, 2, –3) and perpendicular to the planes $3x + y - 2z = 5$ and $2x - 5y - z = 7$, is:

(1) $3x - 10y - 2z + 11 = 0$ (2) $6x - 5y - 2z - 2 = 0$

(3) $11x + y + 17z + 38 = 0$ (4) $6x - 5y + 2z + 10 = 0$

Ans. (3)

Sol. Normal vector of required plane is $\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 2 & -5 & -1 \end{vmatrix} = -11\hat{i} - \hat{j} - 17\hat{k}$

$\therefore 11(x - 1) + (y - 2) + 17(z + 3) = 0$

$11x + y + 17z + 38 = 0$

4. A man is walking on a straight line. The arithmetic mean of the reciprocals of the intercepts of this line on the coordinate axes is $\frac{1}{4}$. Three stones A, B and C are placed at the points (1, 1), (2, 2) and (4, 4) respectively. Then which of these stones is/are on the path of the man?

(1) B only (2) A only (3) All the three (4) C only

Ans. (1)

Sol. $\frac{x}{a} + \frac{y}{b} = 1$

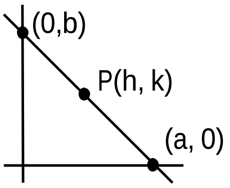
$\frac{h}{a} + \frac{k}{b} = 1$ ....(1)

$\frac{\frac{1}{a} + \frac{1}{b}}{2} = \frac{1}{4}$

$\therefore \frac{1}{a} + \frac{1}{b} = \frac{1}{2}$ ....(ii)

$\therefore$ Line passes through fixed point B(2, 2)

(from (1) and (2))



5. The statement among the following that is a tautology is:

(1) $A \wedge (A \vee B)$ (2) $B \to [A \wedge (A \to B)]$ (3) $A \vee (A \wedge B)$ (4) $[A \wedge (A \to B)] \to B$

Ans. (4)

Sol. $A \wedge (\sim A \vee B) \to B$

$= [(A \wedge \sim A) \vee (A \wedge B)] \to B$

$= (A \wedge B) \to B$

$= \sim A \vee \sim B \vee B$

$= t$

6. Let $f : R \to R$ be defined as $f(x) = 2x-1$ and $g : R - \{1\} \to R$ be defined as $g(x) = \dfrac{x - \frac{1}{2}}{x - 1}$.

Then the composition function $f(g(x))$ is :

(1) both one-one and onto (2) onto but not one-one

(3) neither one-one nor onto (4) one-one but not onto

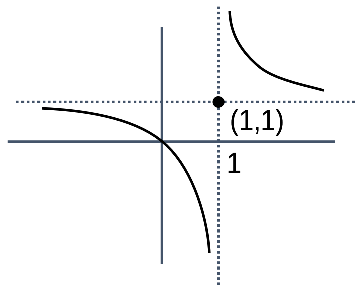
Ans. (4)

Sol. $f(g(x)) = 2g(x) - 1$

$= 2\dfrac{\left(x - \frac{1}{2}\right)}{x - 1} = \dfrac{x}{x - 1}$

$f(g(x)) = 1 + \dfrac{1}{x - 1}$

one-one, into



7. If f:R $\rightarrow$ R is a function defined by f(x) = [x–1] $\cos\left(\frac{2x-1}{2}\right)\pi$, where [.] denotes the greatest integer function, then f is :

(1) discontinuous only at x = 1

(2) discontinuous at all integral values of x except at x = 1

(3) continuous only at x = 1

(4) continuous for every real x

Ans. (4)

Sol. Doubtful points are x = n, n $\in$ I

$$\text{L.H.L} = \lim_{x\to n^-}[x-1]\cos\left(\frac{2x-1}{2}\right)\pi = (n-2)\cos\left(\frac{2n-1}{2}\right)\pi = 0$$

$$\text{R.H.L} = \lim_{x\to n^+}[x-1]\cos\left(\frac{2x-1}{2}\right)\pi = (n-1)\cos\left(\frac{2n-1}{2}\right)\pi = 0$$

f(n) = 0

Hence continuous.

8. The function $f(x) = \frac{4x^3-3x^2}{6} - 2\sin x + (2x-1)\cos x$:

(1) increases in $\left[\frac{1}{2},\infty\right)$

(2) decreases $\left(-\infty,\frac{1}{2}\right]$

(3) increases in $\left(-\infty,\frac{1}{2}\right]$

(4) decreases $\left[\frac{1}{2},\infty\right)$

Ans. (1)

Sol. f'(x) = (2x – 1) (x –sinx)

$\Rightarrow f'(x) \geq 0$ in $x \in \left[\frac{1}{2},\infty\right)$

and $f'(x) \leq 0$ in $x \in \left(-\infty,\frac{1}{2}\right]$

9. The distance of the point (1, 1, 9) from the point of intersection of the line $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2}$ and the plane x + y + z = 17 is:

(1) $\sqrt{38}$ (2) $19\sqrt{2}$ (3) $2\sqrt{19}$ (4) 38

Ans. (1)

Sol. $\dfrac{x-3}{1}=\dfrac{y-4}{2}=\dfrac{z-5}{2}=\lambda$

$\Rightarrow x = \lambda+3,\ y = 2\lambda+4,\ z = 2\lambda+5$

Which lines on given plane hence

$\Rightarrow \lambda + 3 + 2\lambda + 4 + 2\lambda + 5 = 17$

$\Rightarrow \lambda=\dfrac{5}{5}=1$

Hence, point of intersection is Q (4, 6, 7)

$\therefore$ Required distance = PQ

$= \sqrt{9+25+4}$

$= \sqrt{38}$

10. $\lim\limits_{x\to 0}\dfrac{\int_0^{x^2}\left(\sin\sqrt{t}\right)dt}{x^3}$ is equal to :

(1) $\dfrac{2}{3}$ (2) 0 (3) $\dfrac{1}{15}$ (4) $\dfrac{3}{2}$

Ans. (1)

Sol. $\lim\limits_{x\to 0}\dfrac{\int_0^{x^2}\sin\sqrt{t}\,dt}{x^3}=\lim\limits_{x\to 0}\dfrac{\left(\sin|x|\right)2x}{3x^2}=\lim\limits_{x\to 0}\left(\dfrac{\sin x}{x}\right)\times\dfrac{2}{3}=\dfrac{2}{3}$

11. Two vertical poles are 150 m apart and the height of one is three times that of the other. If from the middle point of the line joining their feet, an observer finds the angles of elevation of their tops to be complementary, then the height of the shorter pole (in meters) is:

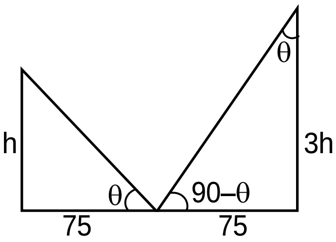
(1) 25 (2) $20\sqrt{3}$ (3) 30 (4) $25\sqrt{3}$

Ans. (4)

Sol. $\tan\theta = \dfrac{h}{75}=\dfrac{75}{3h}$

$\Rightarrow h^2 = \dfrac{(75)^2}{3}$

$h = 25\sqrt{3}\text{m}$



12. If the tangent to the curve $y = x^3$ at the point $P(t, t^3)$ meets the curve again at Q, then the ordinate of the point which divides PQ internally in the ratio 1 : 2 is :

(1) $-2t^3$ (2) $-t^3$ (3) 0 (4) $2t^3$

Ans. (1)

Sol. Equation of tangent at $P(t, t^3)$

$(y - t^3) = 3t^2(x - t)$ ....(1)

Now solve the above equation with

$y = x^3$ ....(2)

By (1) & (2)

$x^3 - t^3 = 3t^2(x - t)$

$x^2 + xt + t^2 = 3t^2$

$x^2 + xt - 2t^2 = 0$

$(x - t)(x + 2t) = 0$

$\Rightarrow x = -2t \Rightarrow Q(-2t, -8t^3)$

Ordinate of required point $= \dfrac{2t^3 + \left(-8t^3\right)}{3} = -2t^3$

13. The area (in sq. units) of the part of the circle $x^2+y^2=36$, which is outside the parabola $y^2=9x$, is :

(1) $24\pi + 3\sqrt{3}$

(2) $12\pi + 3\sqrt{3}$

(3) $12\pi - 3\sqrt{3}$

(4) $24\pi - 3\sqrt{3}$

Ans. (4)

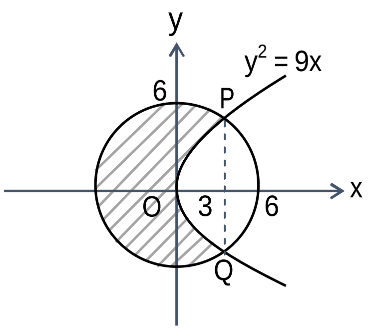
Sol. The curves intersect at point $(3, \pm 3\sqrt{3})$

Required area

$$= \pi r^2 - 2\left[\int_0^3 \sqrt{9x}\,dx + \int_3^6 \sqrt{36 - x^2}\,dx\right]$$

$$= 36\pi - 12\sqrt{3} - 2\left(\frac{x}{2}\sqrt{36 - x^2} + 18\sin^{-1}\left(\frac{x}{6}\right)\right)_3^6$$

$$= 36\pi - 12\sqrt{3} - 2\left(9 - \left(\frac{9\sqrt{3}}{2} + 3\pi\right)\right) = 24\pi - 3\sqrt{3}$$



14. If $\int \frac{\cos x - \sin x}{\sqrt{8 - \sin 2x}}dx = a\sin^{-1}\left(\frac{\sin x + \cos x}{b}\right) + c$, where c is a constant of integration, then the ordered pair (a, b) is equal to :

(1) (1, –3) (2) (1, 3) (3) (–1, 3) (4) (3, 1)

Ans. (2)

Sol. put sin x + cos x = t $\Rightarrow$ 1 + sin 2x = t^2

$\Rightarrow$ (cos x – sin x) dx = dt

$$\therefore I = \int \frac{dt}{\sqrt{8 - (t^2 - 1)}} = \int \frac{dt}{\sqrt{9 - t^2}} = \sin^{-1}\left(\frac{t}{3}\right) + C = \sin^{-1}\left(\frac{\sin x + \cos x}{3}\right) + C$$

$\Rightarrow$ a = 1 and b = 3

15. The population $P = P(t)$ at time 't' of a certain species follows the differential equation $\frac{dP}{dt} = 0.5P - 450$. If $P(0) = 850$, then the time at which population becomes zero is :

(1) $\frac{1}{2}\log_e 18$ (2) $2\log_e 18$ (3) $\log_e 9$ (4) $\log_e 18$

Ans. **(2)**

Sol. $\frac{dp}{dt} = \frac{p-900}{2}$

$$\int_{850}^{0} \frac{dp}{p-900} = \int_{0}^{t} \frac{dt}{2}$$

$$\ell n\left|P-900\right|\Big|_{850}^{0} = \frac{t}{2}$$

$$\ell n|900| - \ell n|50| = \frac{t}{2}$$

$$\frac{t}{2} = \ell n|18|$$

$$\Rightarrow t = 2\ell n 18$$

16. The value of

$-{}^{15}C_1 + 2.{}^{15}C_2 - 3.{}^{15}C_3 + ... -15.{}^{15}C_{15} + {}^{14}C_1 + {}^{14}C_3 + {}^{14}C_5 + ... + {}^{14}C_{11}$ is:

(1) 2^{14} (2) $2^{13} - 13$ (3) $2^{16} - 1$ (4) $2^{13} - 14$

Ans. **(4)**

Sol. $S_1 = -{}^{15}C_1 + 2.{}^{15}C_2 - .... - 15\ {}^{15}C_{15}$

$$= \sum_{r=1}^{15}(-1)^r.r.{}^{15}C_r = 15\sum_{r=1}^{15}(-1)^r\ {}^{14}C_{r-1}$$

$$= 15\left(-{}^{14}C_0 + {}^{14}C_1 - ....-{}^{14}C_{14}\right) = 15\ (0) = 0$$

$$S_2 = {}^{14}C_1 + {}^{14}C_3 + ...+{}^{14}C_{11}$$

$$= \left({}^{14}C_1 + {}^{14}C_3 + ...+{}^{14}C_{11} + {}^{14}C_{13}\right) - {}^{14}C_{13}$$

$$= 2^{13} - 14$$

$$= S_1 + S_2 = 2^{13} - 14$$

17. An ordinary dice is rolled for a certain number of times. If the probability of getting an odd number 2 times is equal to the probability of getting an even number 3 times, then the probability of getting an odd number for odd number of times is :

(1) $\frac{3}{16}$ (2) $\frac{1}{2}$ (3) $\frac{5}{16}$ (4) $\frac{1}{32}$

Ans. (2)

Sol. P(odd no. twice) = P(even no. thrice)

$\Rightarrow {}^nC_2\left(\frac{1}{2}\right)^n = {}^nC_3\left(\frac{1}{2}\right)^n \Rightarrow n = 5$

Success is getting an odd number then P(odd successes) = P(1) + P(3) + P(5)

$= {}^5C_1\left(\frac{1}{2}\right)^5 + {}^5C_3\left(\frac{1}{2}\right)^5 + {}^5C_5\left(\frac{1}{2}\right)^5$

$= \frac{16}{2^5} = \frac{1}{2}$

18. Let p and q be two positive number such that p + q = 2 and $p^4 + q^4 = 272$. Then p and q are roots of the equation :

(1) $x^2 - 2x + 2 = 0$ (2) $x^2 - 2x + 8 = 0$

(3) $x^2 - 2x + 136 = 0$ (4) $x^2 - 2x + 16 = 0$

Ans. (4)

Sol. $(p^2 + q^2)^2 - 2p^2q^2 = 272$

$((p + q)^2 - 2pq)^2 - 2p^2q^2 = 272$

$16 + 16pq + 2p^2q^2 = 272$

$(pq)^2 - 8pq - 128 = 0$

$pq = \frac{8 \pm 24}{2} = 16, -8$

$pq = 16$

Now

$x^2 - (p + q)x + pq = 0$

$x^2 - 2x + 16 = 0$

19. If $e^{\left(\cos^2 x+\cos^4 x+\cos^6 x+...\infty\right)\log_e 2}$ satisfies the equation $t^2 - 9t + 8 = 0$, then the value of $\frac{2\sin x}{\sin x+\sqrt{3}\cos x}\left(0<x<\frac{\pi}{2}\right)$ is :

(1) $\frac{3}{2}$ (2) $2\sqrt{3}$ (3) $\frac{1}{2}$ (4) $\sqrt{3}$

Ans. (3)

Sol. $e^{\left(\cos^2 x+\cos^4 x+......\infty\right)\ell n2} = 2^{\cos^2 x+\cos^4 x+......\infty}$

$= 2^{\cot^2 x}$

$t^2 - 9t + 8 = 0 \Rightarrow t = 1, 8$

$\Rightarrow 2^{\cot^2 x} = 1, 8 \Rightarrow \cot^2 x = 0, 3$

$0 < x < \frac{\pi}{2} \Rightarrow \cot x = \sqrt{3}$

$\Rightarrow \frac{2\sin x}{\sin x+\sqrt{3}\cos x} = \frac{2}{1+\sqrt{3}\cot x} = \frac{2}{4} = \frac{1}{2}$

20. The system of linear equations

$3x - 2y - kz = 10$

$2x - 4y - 2z = 6$

$x + 2y - z = 5m$

is inconsistent if :

(1) $k = 3,\ m = \frac{4}{5}$ (2) $k \neq 3, m \in R$

(3) $k \neq 3,\ m \neq \frac{4}{5}$ (4) $k = 3,\ m \neq \frac{4}{5}$

Ans. (4)

Sol. $\Delta = \begin{vmatrix} 3 & -2 & -k \\ 1 & -4 & -2 \\ 1 & 2 & -1 \end{vmatrix} = 0$

$3(4 + 4) + 2(-2 + 2) - k(4 + 4) = 0$

$\Rightarrow k = 3$

$\Delta_x = \begin{vmatrix} 10 & -2 & -3 \\ 6 & -4 & -2 \\ 5m & 2 & -1 \end{vmatrix} \neq 0$

$10(4 + 4) + 2(-6 + 10m) - 3(12 + 20m) \neq 0$

$80 - 12 + 20m - 36 - 60m \neq 0$

$40m \neq 32 \Rightarrow m \neq \frac{4}{5}$

$$\Delta_y = \begin{vmatrix} 3 & 10 & -3 \\ 2 & 6 & -2 \\ 1 & 5m & -1 \end{vmatrix} \neq 0$$

$3(-6 + 10m) - 10(-2 + 2) - 3(10m - 6) \neq 0$

$-18 + 30m - 30m + 18 \neq 0 \Rightarrow 0$

$$\Delta_z = \begin{vmatrix} 3 & -2 & 10 \\ 2 & -4 & 6 \\ 1 & 2 & 5m \end{vmatrix} \neq 0$$

$3(-20m - 12) + 2(10m - 6) + 10(4 + 4) - 40m + 32 \neq 0 \Rightarrow m \neq \frac{4}{5}$

Section – B

1. Let $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$, where $\alpha \in R$. Suppose $Q = [q_{ij}]$ is a matrix satisfying $PQ = kI_3$ for some non-zero $k \in R$. If $q_{23} = -\frac{k}{8}$ and $|Q| = \frac{k^2}{2}$, then $\alpha^2 + k^2$ is equal to ______

Ans. 17

Sol. As $PQ = KI \Rightarrow Q = kP^{-1}I$

now $Q = \frac{k}{|P|}(adjP)\, I \Rightarrow Q = \frac{k}{(20+12\alpha)}\begin{bmatrix} - & - & - \\ - & - & (-3\alpha - 4) \\ - & - & - \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\because q_{23} = \frac{-k}{8} \Rightarrow \frac{k}{(20+12\alpha)}(-3\alpha - 4) = \frac{-k}{8} \Rightarrow 2(3\alpha + 4) = 5 + 3\alpha$

$3\alpha = -3 \Rightarrow \alpha = -1$

also $|Q| = \frac{k^3|I|}{|P|} \Rightarrow \frac{k^2}{2} = \frac{k^3}{(20+12\alpha)}$

$(20+12\alpha) = 2k \Rightarrow 8 = 2k \Rightarrow k = 4$

2. Let B_i(i=1, 2, 3) be three independent events in a sample space. The probability that only B_1 occur is α, only B_2 occurs is β and only B_3 occurs is γ. Let p be the probability that none of the events B_i occurs and these 4 probabilities satisfy the equations $(\alpha - 2\beta)p = \alpha\beta$ and $(\beta - 3\gamma)p = 2\beta\gamma$ (All the probabilities are assumed to lie in the interval (0, 1)). Then $\frac{P(B_1)}{P(B_3)}$ is equal to ____

Ans. **6**

Sol. Let x, y, z be probability of B_1, B_2, B_3 respectively

$\Rightarrow x(1 - y)(1 - z) = \alpha$

$\Rightarrow y(1 - x)(1 - z) = \beta$

$\Rightarrow z(1 - x)(1 - y) = \gamma$

$\Rightarrow (1 - x)(1 - y)(1 - z) = p$

$(\alpha - 2\beta)p = \alpha\beta$

$(x(1-y)(1-z) - 2y(1-x)(1-z))(1-x)(1-y)(1-z) = xy(1-x)(1-y)(1-z)$

$x - xy - 2y + 2xy = xy$

$x = 2y$ ...(1)

Similarly $(\beta - 3r)p = 2\beta r$

$\Rightarrow y = 3z$ ...(2)

From (1) & (2)

$x = 6z$

Now

$\frac{x}{z} = 6$

3. The minimum value of α for which the equation $\frac{4}{\sin x} + \frac{1}{1 - \sin x} = \alpha$ has at least one solution in $\left(0, \frac{\pi}{2}\right)$ is _____

Ans. **9**

Sol. $f(x) = \frac{4}{\sin x} + \frac{1}{1 - \sin x}$

Let sinx = t $\because x \in \left(0, \frac{\pi}{2}\right) \Rightarrow 0 < t < 1$

$$f(t) = \frac{4}{t} + \frac{1}{1-t}$$

$$f'(t) = \frac{-4}{t^2} + \frac{1}{(1-t)^2}$$

$$= \frac{t^2 - 4(1-t)^2}{t^2(1-t)^2}$$

$$= \frac{(t-2(1-t))(t+2(1-t))}{t^2(1-t)^2}$$

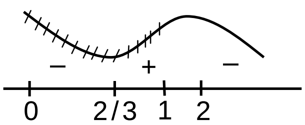
$$= \frac{(3t-2)(2-t)}{t^2(1-t)^2}$$

f_{min} at $t = \frac{2}{3}$

$$\alpha_{min} = f\left(\frac{2}{3}\right) = \frac{4}{\frac{2}{3}} + \frac{1}{1-\frac{2}{3}}$$

$= 6 + 3$

$= 9$



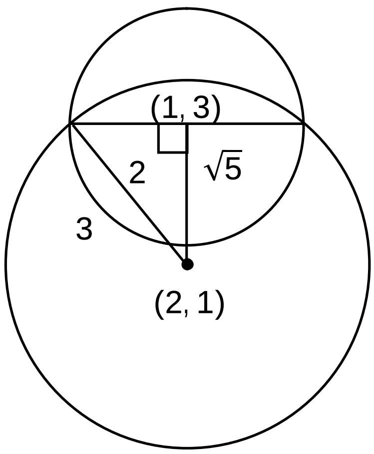
4. If one of the diameters of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord of another circle 'C' whose center is at (2,1), then its radius is ______

Ans. **3**

distance between (1, 3) and (2, 1) is $\sqrt{5}$

$\therefore \left(\sqrt{5}\right)^2 + (2)^2 = r^2$

$\Rightarrow r = 3$



5. $\lim\limits_{x\to\infty} \tan\left\{\sum\limits_{r=1}^{n} \tan^{-1}\left(\frac{1}{1+r+r^2}\right)\right\}$ is equal to ______

Ans. **1**

Sol.
$$\tan\left(\lim_{n\to\infty}\sum_{r=1}^{n}\left[\tan^{-1}(r+1)-\tan^{-1}(r)\right]\right)$$
$$= \tan\left(\lim_{n\to\infty}\left(\tan^{-1}(n+1)-\frac{\pi}{4}\right)\right)$$
$$= \tan\left(\frac{\pi}{4}\right) = 1$$

6. If $\int\limits_{-a}^{a}(|x|+|x-2|)\,dx = 22$,(a > 2) and [x] denotes the greatest integer $\le$ x, then $\int\limits_{a}^{-a}(x+[x])\,dx$ is equal to _____

Ans. **3**

Sol.
$$\int_{-a}^{0}(-2x+2)\,dx+\int_{0}^{2}(x+2-x)\,dx+\int_{2}^{a}(2x-2)\,dx = 22$$
$$x^2-2x\,|_0^{-a} + 2x\,|_0^2 + x^2-2x\,|_2^a = 22$$
$$a^2 + 2a + 4 + a^2 - 2a - (4-4) = 22$$
$$2a^2 = 18 \Rightarrow a = 3$$
$$\int_{3}^{-3}(x+[x])\,dx = -\left(\int_{-3}^{3}(x+[x])\,dx\right) = -\left(\int_{-3}^{3}[x]dx\right)$$
$$= -(-3-2-1+0+1+2) = 3$$

7. Let three vectors $\vec{a},\vec{b}$ and $\vec{c}$ be such that $\vec{c}$ is coplanar with $\vec{a}$ and $\vec{b}$, $\vec{a}.\vec{c}=7$ and $\vec{b}$ is perpendicular to $\vec{c}$, where $\vec{a}=-\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=2\hat{i}+\hat{k}$, then the value of $2\left|\vec{a}+\vec{b}+\vec{c}\right|^2$ is _______

Ans. **75**

Sol.
$$\vec{c} = \lambda\left(\vec{b}\times\left(\vec{a}\times\vec{b}\right)\right)$$
$$= \lambda\left(\left(\vec{b}.\vec{b}\right)\vec{a}-\left(\vec{b}.\vec{a}\right)\vec{b}\right)$$
$$= \lambda\left(5\left(-\hat{i}+\hat{j}+\hat{k}\right)+2\hat{i}+\hat{k}\right)$$

$= \lambda\left(-3\hat{i}+5\hat{j}+6\hat{k}\right)$

$\vec{c}.\vec{a} = 7 \Rightarrow 3\lambda + 5\lambda + 6\lambda = 7$

$\lambda = \frac{1}{2}$

$\therefore 2\left|\left(\frac{-3}{2}-1+2\right)\hat{i}+\left(\frac{5}{2}+1\right)\hat{j}+(3+1+1)\hat{k}\right|^2$

$= 2\left(\frac{1}{4}+\frac{49}{4}+25\right) = 25+50 = 75$

8. Let $A = \{n \in N : n \text{ is a 3-digit number}\}$

$B = \{9k + 2 : k \in N\}$

and $C : \{9k + \ell : k \in N\}$ for some ℓ $(0 < \ell < 9)$

If the sum of all the elements of the set $A \cap (B \cup C)$ is 274×400, then ℓ is equal to __

Ans. **5**

Sol. 3 digit number of the form 9K + 2 are {101, 109, ...992}

$\Rightarrow$ Sum equal to $\frac{100}{2}$ (1093) $= s_1 = 54650$

$274 \times 400 = s_1 + s_2$

$274 \times 400 = \frac{100}{2}[101 + 992] + s_2$

$274 \times 400 = 50 \times 1093 + s_2$

$s_2 = 109600 - 54650$

$s_2 = 54950$

$s_2 = 54950 = \frac{100}{2}\left[(99+\ell)+(990+\ell)\right]$

$1099 = 2\ell + 1089$

$\ell = 5$

9. If the least and the largest real values of α, for which the equation $z+\alpha|z-1|+2i=0$ $\left(z \in C \text{ and } i=\sqrt{-1}\right)$ has a solution, are p and q respectively; then $4(p^2 + q^2)$ is equal to ___

Ans. **10**

Sol. $x + iy + \alpha\sqrt{(x-1)^2+y^2}+2i=0$

$\therefore y + 2 = 0$ and $x + \alpha\sqrt{(x-1)^2+y^2}=0$

$y = -2$ & $x^2 = \alpha^2(x^2 - 2x + 1 + 4)$

$\alpha^2 = \dfrac{x^2}{x^2-2x+5} \Rightarrow x^2(\alpha^2-1)-2x\alpha^2+5\alpha^2 = 0$

$x \in R \Rightarrow D \geq 0$

$4\alpha^4 - 4(\alpha^2-1)5\alpha^2 \geq 0$

$\alpha^2\left[4\alpha^2-2\alpha^2+20\right] \geq 0$

$\alpha^2\left[-16\alpha^2+20\right] \geq 0$

$\alpha^2\left[\alpha^2-\dfrac{5}{4}\right] \leq 0$

$0 \leq \alpha^2 \leq \dfrac{5}{4}$

$\therefore \alpha^2 \in \left[0, \dfrac{5}{4}\right]$

$\therefore \alpha \in \left[-\dfrac{\sqrt{5}}{2}, \dfrac{\sqrt{5}}{2}\right]$

then $4[(q)^2+(p)^2] = 4\left[\dfrac{5}{4}+\dfrac{5}{4}\right] = 10$

10. Let M be any 3×3 matrix with entries from the set {0, 1, 2}. The maximum number of such matrices, for which the sum of diagonal elements of M^TM is seven, is __

Ans. **540**

Sol. $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$

$a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2 + h^2 + i^2 = 7$

Case I : Seven (1's) and two (0's)

${}^9C_2 = 36$

Case II : One (2) and three (1's) and five (0's)

$\frac{9!}{5!3!} = 504$

$\therefore$ Total = 540